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Performance evaluation of gamma-type Stirling engine using combined Schmidt and mechanical loss model

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ABSTRACT

This paper focuses on the study of gamma-type Stirling engine prototype using a combined Schmidt closed-form and mechanical loss analysis. Not restricted to optimizing the indicated power as classic Schmidt theory is set to, this analysis allows to maximize the shaft power due to the mechanical loss in power transmission. For this purpose, MATLAB code was developed to calculate the indicated and the shaft powers of the engine at different operating parameters. The results showed that shaft power peaks at swept volume ratios smaller than those of indicated power at different values of mechanism effectiveness. Within the range of engine mechanism effectiveness typically between 0.7 and 0.9, it was found that maximum shaft power for this particular engine can be achieved at different optimum values of swept volume ratio between 0.75 and 0.95 and phase angle between 80° and 90°. However, an optimum swept volume ratio was found to be k=0.55 of the same engine size for different scenarios of operation. Also, the developed model can be used as a design tool in the preliminary stage to find the optimum geometry of the engine. The new engine design parameters including the stroke, the crank radius and power piston bore, and engine alteration were presented.

Keywords: evaluation, gamma-type, mechanical-loss model, performance, stirling engine, Schmidt

INTRODUCTION

Stirling engine is one of the heat engines that have the potential to harvest renewable heat sources (biomass, solar, and geothermal) for electricity generation (Organ, 2013). Solar dish-Stirling system is a maturing technology in this field (Singh & Kumar, 2018). On the other hand, Stirling engine has a great potential to recover heat from low-grade heat sources such as geothermal or waste heat with typical temperatures between 100 °C and 200 °C (Wang et al., 2016). Its regenerative thermodynamic cycle, composed of isothermal expansion, compression, and isochoric heating and cooling, is theoretically efficient. However, the existence of irreversibility dramatically degrades the engine performance. Descending from simple to complex, analysis methods of zeroth-, first-, second-, third-, and forth-order exist in the literature. An estimate of shaft power for most documented Stirling engines can be obtained from Beale's empirical formula, which is known as zero-order analysis.

A closed-form set of equations was introduced as a first-order approach by Schmidt (1871) with the assumption of isothermal heat exchange in engine working spaces and sinusoidal volume variation. The second-order analysis stems from ideal adiabatic model solving for conservation of mass and energy equations inside five control volumes of the engine with decoupling the losses. The third-order approach is a nodal analysis solves conservation equations in one dimension and time. The fourth-order analysis solves the complex transport equations using computational fluid dynamics (CFD).

Among the three mechanical layouts of kinematic Stirling engine (alpha, beta, and gamma), beta and gamma-type are widely analyzed compared to alpha. Stirling engine ST05G-CNC is a gamma-type prototype that was designed and opened for academic research and optimization. This engine is classified as a high temperature difference gamma-type with maximum heater temperature of 650 °C and a nominal shaft power of 500 W. Several studies were conducted to analyses and improve the performance of this particular engine. Starting from second-order analysis, Alfarawi et al. (2016b) introduced a modified non-ideal adiabatic model to maximize engine shaft power by suggesting helium as a working fluid instead of nitrogen and lowering the cooling temperature when utilizing the cryogenic energy storage. Hooshang et al. (2015) optimized the engine parameters of displacer stroke, phase angle, and frequency for maximum power using neural network optimization based on third-order analysis. Alfarawi et al. (2016a) adopted comprehensive two-dimensional CFD approach to analyze and improve the engine performance. Optimum values of phase angle and dead volume (connecting

Table 1. List of parameters used in Schmidt (1871) analysis

Parameter/unit	Definition
n [rpm]	Engine speed
$\omega = 2\pi n/60$ [rad/s]	Angular velocity
V_1 [cm^3]	Displacer swept volume
V_2 [cm^3]	Piston swept volume
V_T=V_1+V_2 [cm^3]	Total swept volume
$k = V_2/V_1$ [-]	Swept volume ratio
$\bar{P} = \sqrt{P_{max}P_{min}}$ [bar]	Cyclic pressure
P_b=$\bar{P}$	External buffer pressure
V_D [cm^3]	Dead volume
$\chi = V_D/V_1$ [-]	Dead volume ratio
T_E [K]	Hot space temperature
T_C [K]	Cold space temperature
$\tau = T_C/T_E$ [-]	Ttemperature ratio
$T_D = (T_E + T_C)/2$ [K]	Dead space temperature
α [rad]	Phase angle
E [%]	Mechanism effectiveness

pipe) were obtained for maximum indicated power. Kuban et al. (2019) conducted a computationally expensive three-dimensional CFD analysis to deeply understand the fluid flow and heat transfer characteristics inside the same engine. For efficient energy conversion, Stirling engine has to be properly optimized in terms of power and efficiency. To fill the gap in literature about optimizing the performance of this particular engine at different scenarios of operation. The aim of this work; first is to analyze this particular engine with a simpler approach using combined Schmidt and mechanical loss model in order to characterize the optimum design parameters for the same engine size such as swept volume ratio and phase angle for maximum shaft power. Second is to evaluate engine performance at different scenarios of operation such as when the engine is utilized with high and low-grade heat source.

METHOD

One of the classical methods widely used for the analysis of Stirling engines is Schmidt model (Schmidt, 1871). Recalling that main assumptions made in this analysis are, as follows:

1. Compression and expansion processes are taking place isothermally.
2. The working gas is treated as an ideal gas.
3. The volume variation is sinusoidal.
4. The instantaneous pressure throughout the engine spaces is uniform with no leakage of working gas.

In terms of engine operating and geometrical parameters, the following parameters can be defined in the analysis, as shown in **Table 1**. Full derivation procedure of Schmidt model can be found in the work of Senft (2002). However, the main equations of Schmidt model are represented here. The total volume variation of the engine can be calculated, as follows:

$$V = \frac{V_T}{\kappa+1}\left(1 + \frac{\kappa}{2}(1 + \cos(\omega t)) + \chi\right).$$

Also, the cyclic pressure is determined, as follows:

$$p = \frac{\bar{P}\sqrt{Y^2 - X^2}}{Y + X\cos(\omega t - \theta)},$$

where

$$Y = 1 + \tau + k + \frac{4\chi\tau}{1+\tau},$$

$$X = \sqrt{k^2 - 2k(1-\tau)\cos\alpha + (1-\tau)^2}, \text{ and}$$

$$\theta = \cos^{-1}\left(\frac{k - (1-\tau)\cos\alpha}{X}\right).$$

Closed-form formula of indicated work, W is, as follows:

$$W = \frac{\pi(1-\tau)V_T\bar{P}k\sin\alpha}{(k+1)\left(\sqrt{Y^2 - X^2} + Y\right)}.$$

Following the assumption that buffer pressure equals the mean workspace pressure for most Stirling engines. The forced work is obtained from the compression work done on the piston when the workspace pressure is above the buffer pressure, plus the expansion work done by the piston when the workspace pressure is below the buffer pressure. The forced work per cycle, $\bar{W}$, can be evaluated, as follows:

$$\bar{W} = \oint[(p - p_b)\,dV]^{-}.$$

The mechanical work of a reciprocating engine is transferred to the shaft through the piston and flywheel in both directions. The work transfer is partly degraded by frictional losses in these mechanical parts depending on mechanism effectiveness. Therefore, for a constant effectiveness of the engine mechanism between $0 < E \leq 1$, the shaft power, W_s can be calculated, as follows:

$$W_s = EW - (1/E - E)\bar{W}.$$

The mechanical efficiency, η_m can be calculated in terms of shaft work and indicated work, as follows:

$$\eta_m = \frac{W_s}{W}.$$

The flow chart of the developed algorithm, written in MATLAB environment, is provided in **Figure 1**. The input parameters used in MATLAB code are for gamma-type Stirling engine prototype developed at the University of Birmingham and its data are presented in **Table 2** (Alfarawi et al., 2016b).

RESULTS

Model Verification

Figure 2 shows the shape of PV diagram in one cycle of Stirling engine obtained from the present model. PV diagram was compared with the results of Alfarawi et al. (2016a) at the same operating conditions. Good agreement was found in terms of the upper and lower limits of the instantaneous pressure. A closer look at the calculated indicated powers from PV diagrams, the present model reads 1,073 W, which is 17.0% higher. The current analysis is more idealized and not intended to predict the actual performance of the engine. However, trends can be easily generalized with good confidence from this analysis.

Effect of Swept Volume Ratio

In this section, the gamma Stirling engine under study was simulated using the present model to see how swept volume ratio affects the indicated and shaft powers at different values of mechanism effectiveness. It should be noted that the engine size is kept constant at 969 cm^3 for all cases. This ensures finding the optimum swept volume ratio between piston and displacer sections for maximum power.

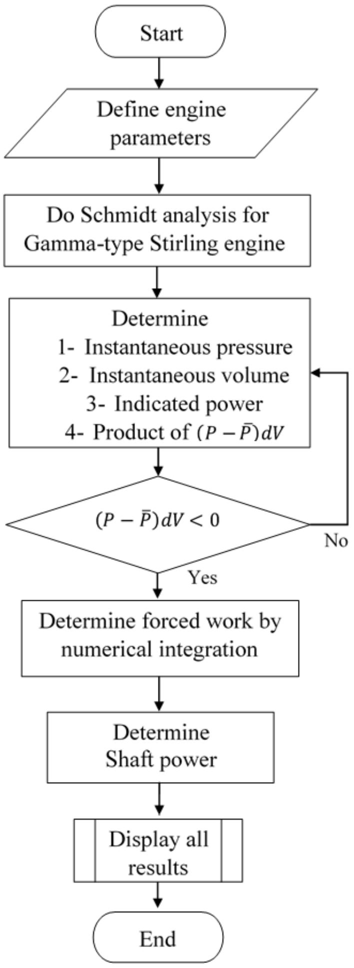

Figure 1. Flow chart of MATLAB code (Source: Authors' own elaboration)

Table 2. Input parameters for MATLAB code

Parameter	Value/unit
n	500 [rpm]
V_1	543 [cm^3]
V_2	426 [cm^3]
V_D	886 [cm^3]
T_E	650 [°C]
T_C	15 [°C]
T_D	332.5 [°C]
$P_b=\bar{P}$	10 [bar]
ω	52.35 [rad/s]
α	π/2 [rad]
τ	0.31 [-]
k	0.7845 [-]
χ	1.63 [-]
E	0.7 [-]

As shown in **Figure 3**, it can be seen that at constant mechanism effectiveness E=0.7, the shaft power optimum occurs at a smaller swept volume ratio (k=0.75) compared to that of indicated power optimum at (k=1.20).

As the mechanism effectiveness increased from 0.7 to 0.8 as indicated in **Figure 4**, the same behavior of shaft power is obtained noting that indicated power is independent of mechanism effectiveness. However, the shaft power peaks at a swept volume ratio (k=0.85) compared to that of indicated power (k=1.20). Shaft work is a function of buffer pressure, PV shape, and mechanism effectiveness.

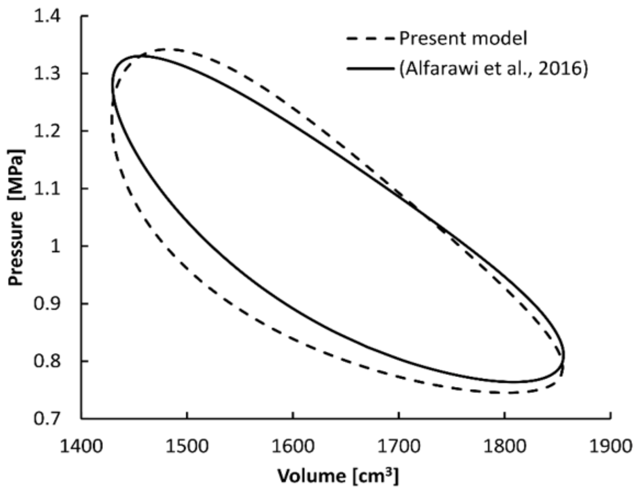

Figure 2. Indicated PV diagram (Source: Authors' own elaboration)

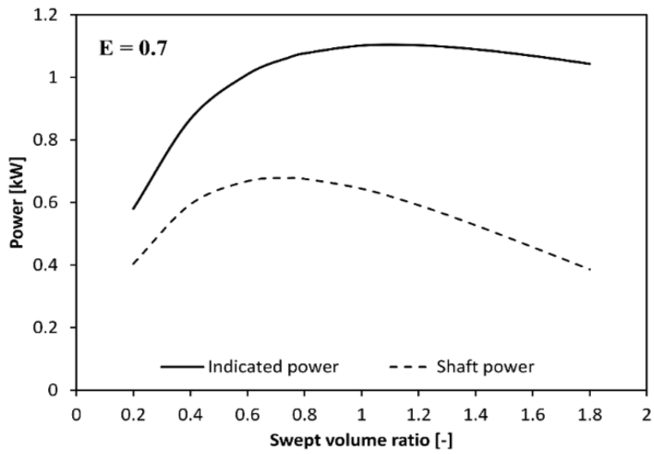

Figure 3. Swept volume ratio effect on engine power at E=0.7 (Source: Authors' own elaboration)

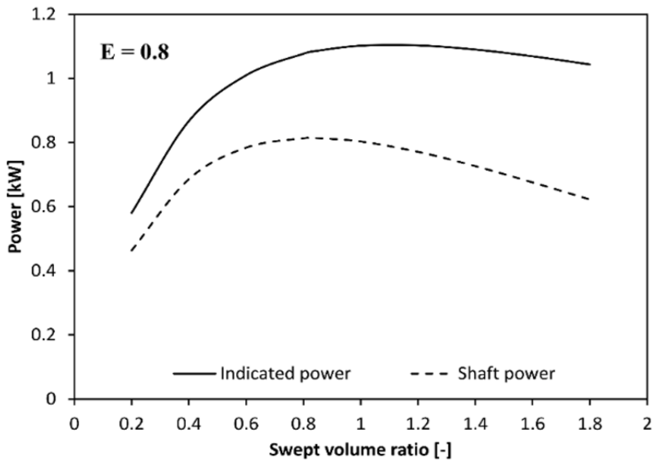

Figure 4. Swept volume ratio effect on engine power at E=0.8 (Source: Authors' own elaboration)

With increasing the mechanism effectiveness from 0.8 to 0.9, as shown in **Figure 5**, the indicated power and shaft power curves become closer due to the effective mechanism of transmitting the power output.

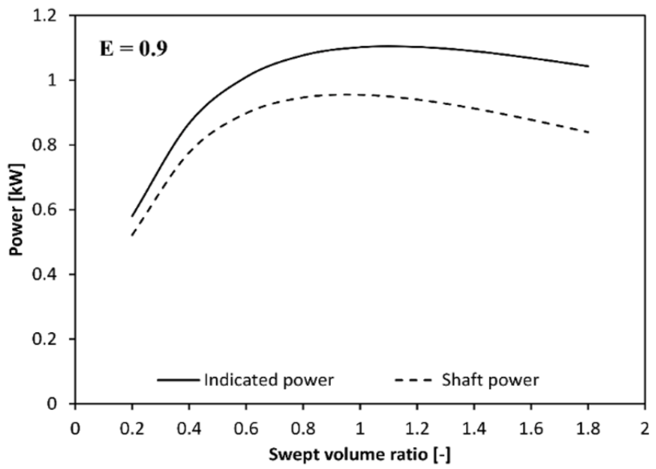

Figure 5. Swept volume ratio effect on engine power at *E*=0.9 (Source: Authors' own elaboration)

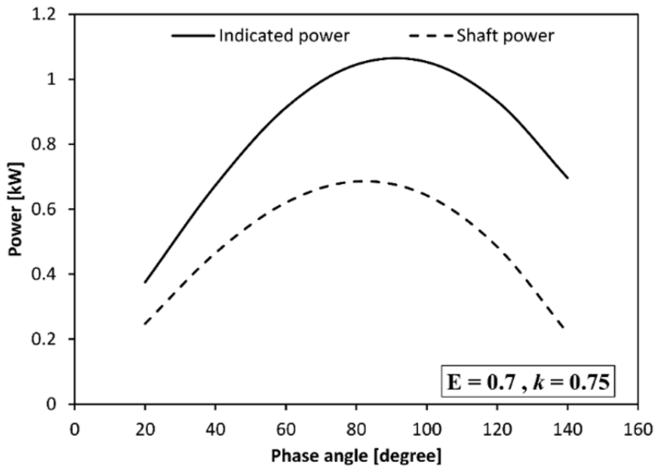

Figure 6. Phase angle effect on engine power at *k*=0.75 (Source: Authors' own elaboration)

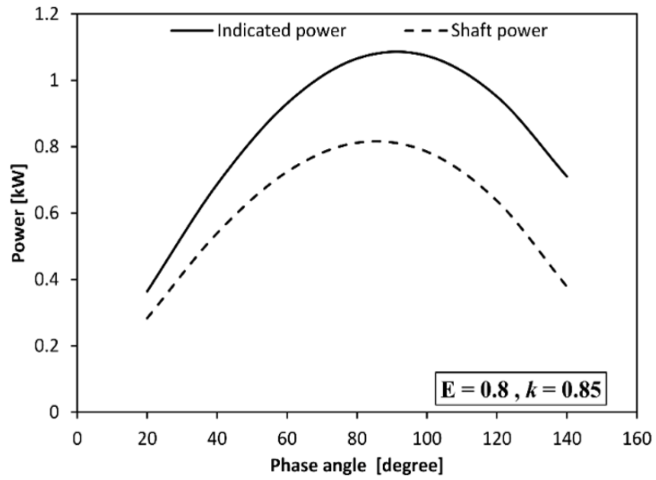

Figure 7. Phase angle effect on engine power at *k*=0.85 (Source: Authors' own elaboration)

Swept volume ratios for maximum indicated and shaft powers read 1.20 and 0.95, respectively. Interesting point to be noticed is that in all cases swept volume ratio at which shaft power peaks is very close to the mechanism effectiveness.

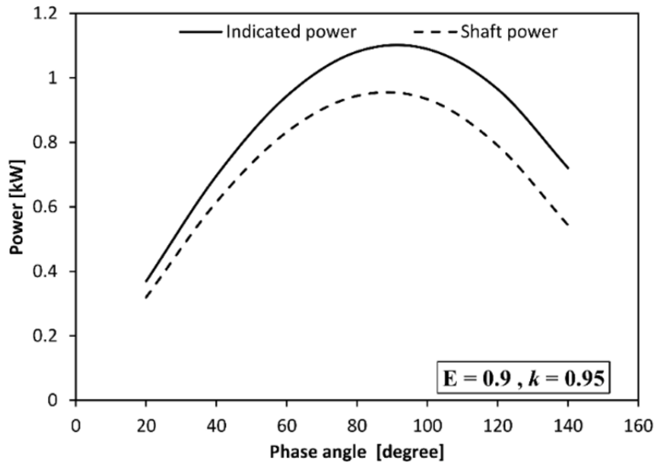

Figure 8. Phase angle effect on engine power at *k*=0.95 (Source: Authors' own elaboration)

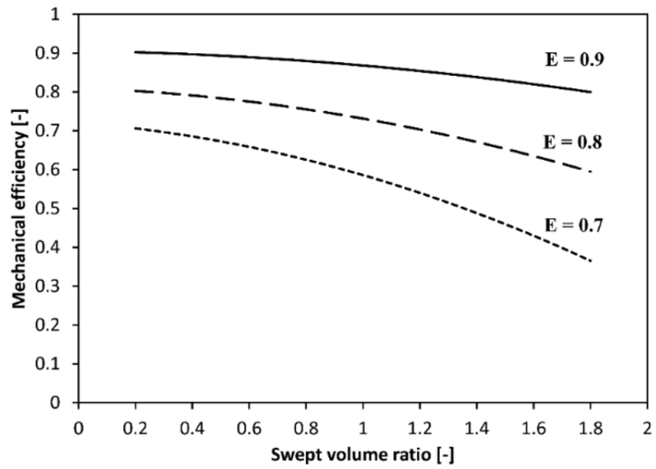

Figure 9. Mechanical efficiency variation with swept volume ratio at different mechanism effectiveness values (Source: Authors' own elaboration)

Effect of Phase Angle

The phase angle is one of the control means of engine power and is normally fixed to be 90° for most Stirling engines. But effect of phase angle on engine performance at different optimum values of swept volume ratios is represented. At the optimum swept volume ratio (*k*=0.75), as shown in **Figure 6**, the maximum shaft power occurs at a phase angle of 80° rather than the maximum indicated power does at 90°.

With increasing mechanism effectiveness to 0.8 and at an optimum swept volume ratio (*k*=0.85), as shown in **Figure 7**, maximum shaft power occurs normally at a phase angle of 90°.

Again, the same results were obtained when increasing the mechanism effectiveness to 0.9 at an optimum swept volume ratio (*k*=0.95), as shown in **Figure 8**.

Mechanical Efficiency Results

Higher mechanical efficiency of Stirling engine is favorable for more reliability and durability. The variation of mechanical efficiency with swept volume ratio at different values of mechanism effectiveness is shown in **Figure 9**. Smaller swept volume ratios tend to improve mechanical efficiency

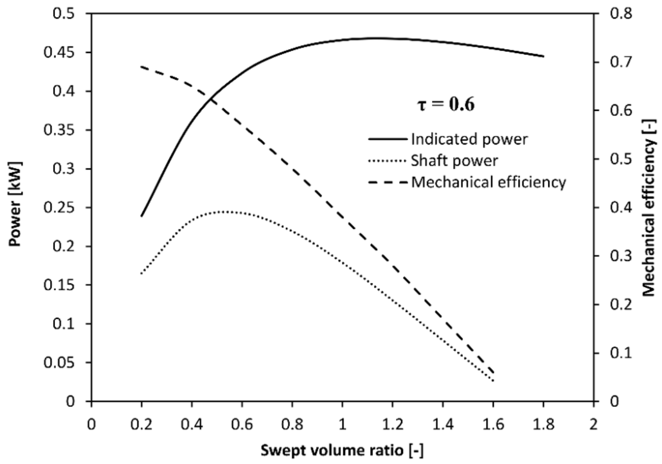

Figure 10. Swept volume ratio effect on engine power and mechanical efficiency at E=0.7 & τ=0.6 (Source: Authors' own elaboration)

approaching mechanism effectiveness. But swept volume ratio that yields maximum shaft power is still a good choice.

Effect of Lower Limit of Hot Temperature

To find optimum swept volume ratio for current engine size, power characteristics should be investigated at lower limit of hot space temperature. This means that engine can still produce less useful power when it runs on low-grade heat sources. A temperature of 200 °C was selected, corresponding to a temperature ratio τ=0.6 to avoid any further degradation of engine performance below this temperature as reported in the manufacturer's documentation (Alfarawi et al., 2016b).

Figure 10 shows the effect of swept volume ratio on engine powers at a mechanism effectiveness of 0.7 and temperature ratio of 0.6. As expected, both indicated and shaft powers increase with increasing swept volume ratio up to the optimum point.

The maximum indicated power of 463 W is obtained at a swept volume ratio k=1.40 compared to a maximum shaft power of 244 W at k=0.55. The shaft power curve then starts to decrease at higher swept volume ratios until it stops running after k=1.6. At this point, it is clear that the swept volume ratio for maximum shaft power (k=0.55) departs from that optimum ratio obtained at temperature ratio τ=0.3 as depicted in **Figure 3**, which reads k=0.75. However, if the engine runs with this swept volume ratio k=0.75 at temperature ratio τ=0.6 (low heat source), both shaft power and mechanical efficiency would drop by 7.5% and 16.0%, respectively. On the other hand, if the engine runs with swept volume ratio k=0.55 at temperature ratio τ=0.3 (**Figure 3**), a 3.0% drop in shaft power is compensated by a 4.5% increase in mechanical efficiency. Therefore, the rule of thumb approach is to select the swept volume ratio k=0.55 for the current engine size for the sake of all scenarios of operation.

Practical Implications

Recalling from this analysis that, the optimal swept volume ratio was found to be k=0.55. The new displacer and power piston swept volumes are accordingly found from simulations to be 625 cm^3 and 344 cm^3, respectively. In order to fix theses new swept volumes in the engine, it is not practical to alter the displacer piston bore since it is interconnected with the three heat exchangers (heater, regenerator, and cooler). The only possibility left is to alter with ease the bore of power piston assembly. The new stroke as listed in **Table 3** is found to be 86 mm, which is 15.0% higher than the original stroke and is calculated from the modified displacer swept volume and its fixed bore. The new bore of power piston is found to be 71.4 mm, which is 18.0% smaller than the original bore and is calculated from its modified swept volume and the new stroke. The original height of displacer piston can be practically adjusted to accommodate the new stroke. Due to this modification, the crank radius of the drive mechanism must be increased by 15.0% to accommodate the new engine parameters to be 43.0 mm.

Table 3. Modified parameters of engine layout

Parameter	Original	Modified
Displacer piston swept volume	543 [cm^3]	625 [cm^3]
Power piston swept volume	426 [cm^3]	344 [cm^3]
Stroke	75 [mm]	86 [mm]
Displacer piston bore	95.96 [mm]	Fixed
Power piston bore	87.46 [mm]	71.40 [mm]
Crank radius	37.50 [mm]	43.00 [mm]

CONCLUSIONS

MATLAB code was developed to simulate a gamma-type Stirling engine prototype using Schmidt/mechanical loss analysis. The developed code is a design tool that can be used in the preliminary stage of engine design. The indicated power, shaft power, and mechanical efficiency can be predicted at different operating parameters. The following points can be concluded from this study:

1. Maximum indicated and shaft powers peak at different swept volume ratios.
2. Swept volume ratio at which the shaft power peaks at maximum heater temperature is very close to the mechanism effectiveness.
3. Swept volume ratio at which the shaft power peaks at minimum heater temperature is considerably small.
4. Optimum phase angle for maximum indicated power is 90° and not far from 90° for maximum shaft power.
5. Better mechanical efficiency tends to occur at a smaller swept volume ratio.
6. The optimal swept volume ratio for all scenarios of operation was obtained.
7. The modified bore of power piston was found
8. The modified parameters of crank drive mechanism including the stroke and crank radius were found.

For future work, it is recommended to perform a dynamic analysis on the crank drive mechanism with the new obtained parameters.

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Declaration of interest: No conflict of interest is declared by the authors.

Ethical statement: The authors stated that the study does not require ethical approval since the data used is accessible through existing literature and the programming code used for analysis is developed by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from corresponding author.

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