European Journal of Sustainable Development Research
2026, 10(3), em0398
e-ISSN: 2542-4742
https://www.ejosdr.com/

MODESTUM

Research Article

OPEN ACCESS

Types of ESD-mathematical modelling tasks–Integrating mathematical modelling and education for sustainable development (ESD) for school and teacher education

Sabine Wiegand [1], Rita Borromeo Ferri [1*]

[1] Department of Mathematics and Natural Sciences, Institute of Mathematics, University of Kassel, Kassel, GERMANY
***Corresponding Author:** borromeo@mathematik.uni-kassel.de**

Citation: Wiegand, S., & Borromeo Ferri, R. (2026). Types of ESD-mathematical modelling tasks–Integrating mathematical modelling and education for sustainable development (ESD) for school and teacher education. *European Journal of Sustainable Development Research, 10*(3), em0398. https://doi.org/10.29333/ejosdr/18338

ARTICLE INFO

Received: 23 Oct. 2025

Accepted: 30 Mar. 2026

ABSTRACT

Mathematical education provides a fact-based foundation for addressing challenges related to sustainable development. In this study, sustainability is conceptualized in accordance with the guiding principles of sustainability and education for sustainable development (ESD) formulated by the United Nations (2015). Despite its relevance, research gaps persist regarding the integration of mathematical modelling and ESD in school and teacher education. Task design is central to implementing ESD in mathematics education. Empirical evidence lacks regarding how pre-service teachers develop ESD-modelling-tasks based on specific design-criteria. Thus, firstly this study examines how 35 pre-service mathematics teachers approach to develop ESD-modelling-tasks. Ten participants received explicit instruction in design-criteria for ESD-modelling-tasks. Thus, secondly the goal was to investigate if their developed tasks differ. Qualitative content analysis was used for data analysis. The findings show two approaches to task development and identify three distinct types of ESD-modelling-tasks, providing implications for research and practice.

Keywords: mathematical modelling, education for sustainable development, ESD-modelling-tasks, design-criteria, pre-service mathematics teachers, empirical study

INTRODUCTION

The call for a transformation towards sustainable future issues is not a new trend. Findings from the past decades have prompted politicians to appeal to society to help shape a sustainable future through (individual) actions. This directly raises the question of whether and how we as mathematics educators or the discipline of mathematics education in general can contribute. Within mathematics education many researchers have already dealt extensively with sustainability aspects–taking theoretically reflective positions on the one hand and proposing practical approaches regarding to mathematics teacher education and mathematics teaching on the other (e.g., Barwell, 2013; Coles et al., 2024; Renert, 2011; Skovsmose, 2023). This also becomes evident in the current proceedings of the ICMI-study 27 on mathematics education and the socio-ecological (Le Roux et al., 2025), which offers a broad range and insightful answers for the research in this field, but many open questions remain. In their overview article Makramalla et al. (2025) clarify the origins of the term "socio-ecological", which they use in the context of mathematics education. In doing so, they refer to works, e.g., of Sadler (2022), which brings his ideas further to the science technology-society (STS) movement, so that STS scholar through de Freitas and Sinclair (2014) became aware of mathematics education.

Within a recent systematic review study by Borromeo Ferri and Bulut (2026) 68 studies indexed in the Web of Science and Scopus databases were analyzed to investigate how eucation for sustainable development (ESD) is integrated into mathematics education across different contexts and levels. Findings highlight transformative potential of mathematics education for sustainability and showing participants' greater understanding of real-world issues and stronger engagement.

The authors use the United Nations (2015) framework of sustainability. The focus is on teacher professionalism and task development by connecting mathematical modelling (MM) with the framework of ESD based on "the 2030 agenda for sustainable development 'transforming our world'" with the 17 sustainable development goals (SDGs). In line with the ESD anchored in sub-goal 4.7, the authors pursue the goal of enabling pre-service mathematics teachers (PSMTs) to acquire the "necessary knowledge and qualifications to promote sustainable development" (United Nations, 2015).

MM means, briefly, the solution of real-world problems with the help of mathematical models (Niss & Blum, 2020). Based on previous studies by the authors and other researchers, MM has proven to be a successful practice of integrative teaching and learning of ESD and mathematics (Vorhölter & Siller, 2023; Wiegand & Borromeo Ferri, 2024). However, there are many research gaps regarding the integrated teaching and learning in MM and ESD for schools and teacher education. In particular task formats are crucial for the implementation of ESD in mathematics classrooms. Yet, there is less empirical evidence, how pre-service teachers are able to develop and conduct ESD-modelling tasks along design-criteria and how they deal with this challenge integrated ESD and mathematics, as mentioned above. To shed light on these research gaps was a central aim of our study. Thus, within the university ESD-MM seminars conducted and investigated for two years by the authors, one main goal was that PSMTs learn how mathematics can be the basis to describe, solve and understand sustainability issues for themselves and in their future work in the classroom by specifically developing and implementing ESD-MM tasks based on the theory-based developed design-criteria from the first author (Wiegand, 2024). ESD-modelling tasks include both ESD and modelling conceptualization (Wiegand & Borromeo Ferri, 2023).

In this article, the content of the ESD-MM seminar will be briefly presented in the context of the literature review and previous work of the authors to promote why teacher professionalism is important in order that teachers become and act as 'change agents' (e.g., Singer-Brodowski et al., 2025) for sustainability issues. We will explain the theory-based design-criteria for the ESD-MM tasks. Based on qualitative type-forming qualitative content analysis (Kuckartz, 2016), 35 ESD-MM tasks developed by the PSMTs were analyzed to determine how PSMTs approached the task development process, which is the focus on the first research question (RQ). A part of the sample, 10 PSMTs, were explicitly trained in using the design-criteria. This means that they exercised and applied more deeply the design-criteria along with many best-practice examples, before they developed their ESD-modelling task. Based on this sample of 10 PSMTs, the goal was to examine if differences in the developed tasks can be identified and if, how they can be described. With differences we mean aspects like complexity and opening of the task, chosen SDG-topics and SDG-dimension, focus on learners' age groups and grades or the mathematical content area.

It should be noted that the aim of the study was not to investigate the differences between the PSMT with and without the design-criteria training. Thus, the central RQs of our study were:

RQ1. How did the PSMTs approach the ESD-task development process? (whole sample of 35 PSMTs)

RQ2. The concepts of modelling, the ESD framework and the design-criteria encourage possible differences in the ESD-modelling tasks to be developed, therefore (part of the sample, 10 PSMTs)

a. How do the developed tasks differ?

b. And if, how could they be described?

LITERATURE REVIEW

Sustainable Development Goals, ESD, and Promoting MM in This Context

One way on how sustainability can be conceptualized, which also underlies the authors' study, is the work of the United Nations (2015) SDGs. According to the United Nations (2015) the aim is to achieve a balance between the three dimensions of sustainability–ecological, economic, and social–as well as intra- and intergenerational justice by fulfilling the 17 SDGs. The United Nations framework with the agenda 2030 and the SDGs and the linked framework of ESD, have long been established. Although transformative education (United Nations, 2015) should already be expected to be anchored in school curricula and thus also subject-oriented, like for mathematics, this has not yet been satisfactorily implemented–at least not in Germany. The German Ministry of Education has published an orientation framework for global development learning for secondary and high-schools, based on the United Nations framework (Schreiber & Siege, 2016). For the subject of mathematics and ESD (e.g., Reiss et al., 2016), suggestions are formulated by taking account the core competencies of ESD (de Haan, 2002)–recognizing, evaluating and acting–and the German educational standards in mathematics (Blum et al., 2012). These suggestions are helpful, but regarding research (and practical implementation), there are many open questions, such as how to professionalize mathematics teachers or how to design tasks that ultimately consider both mathematics and ESD and use them in all school forms?

Not limited to Germany, the school subjects of political science, ethics and biology have, in contrast to mathematics, already developed and established ESD content for schools within the framework of the educational approaches of 'global learning' and 'environmental education' (Feinstein & Carlton, 2012; Holst & Brock, 2020).

We see it as a challenge to combine the ESD framework with mathematics education, since ESD has a complex definition. Briefly, ESD means:

(1) incorporating key topics of sustainable development into teaching and learning (e.g., climate change),

(2) using participatory teaching and learning methods that motivate and empower learners to reflect on individual and societal transformation and act for sustainable development, and

(3) promoting competences that enable learners to think critically, imagine future scenarios and make cooperative decisions evoked by decision (making) dilemmas (de Haan, 2002, 2010; UNESCO, 2020).

Due to the inherent interdisciplinary nature of MM, mathematics can be applied in a wide variety of disciplines to answer current real-world questions in a fact-based (e.g., data) and socio-critical way (Borromeo Ferri & Mousoulides, 2017; Kaiser & Sriraman, 2006).

The translation processes between reality and mathematics in the modelling process require modelling competencies (Kaiser, 2007). According to the criteria specified by Maaß (2007), modelling tasks must be realistic,

open, authentic, problem-based and all steps of the modelling process must be passed. MM has been a mandatory competence in the mathematics educational standards for all school types in Germany since 2003. A differentiated approach to mathematical thinking and working is reflected in the requirement levels I to III of a modelling task as well as in the dimensions of mathematics (Blum et al., 2012).

The first requirement level includes the use of familiar and directly recognizable models. Simple mathematical methods are used to represent phenomena from the world of experience. The second requirement level requires several more complex modelling steps. The results of the modelling must be interpreted and assigned to a suitable situation or checked. The third requirement level refers to the highest level of abstraction, which requires the modelling of a complex or unfamiliar situation as well as the reflection and critical assessment of the mathematical models used (Blum et al., 2012). The requirement levels demand knowledge of mathematics and its tools. The Hessian Ministry of Culture describes the dimensions of mathematics as

(1) mathematics in application, as a tool to "understand the environment and cope with the demands of everyday life",

(2) mathematics as a structure with its language, symbols, formulae, and

(3) mathematics as a creative approach to dealing with situations in a self-determined way and solving problems using mathematical means (Hessian Ministry of Education, 2022).

We describe the requirement levels and dimensions as the multi-perspectivity of MM, which presents one of the dimensions of the feature space for data analysis.

Mathematics Teacher Professionalism in Line With the ESD–Framework Related to Modelling

Teacher training programs play an important role in supporting teacher professionalization in tackling sustainable development challenges (Suh & Han, 2019). Results of a study from Moreno-Pino et al. (2021) show that sustainability into higher education mathematics curricula is deemed crucial for fostering professionals' reflective and critical thinking. Nguyen et al. (2020) emphasize that secondary school teachers need to be guided and trained to improve their knowledge and application of contemporary constructivist pedagogical approaches to promote learners' self-direction, collaboration and problem-solving skills about ESD. The authors have implemented this requirement in the ESD-MM seminars with pre-service mathematics secondary school teachers.

Furthermore, in Germany, pre-service secondary and high school teachers must always study two subjects, for example mathematics and physics. This provides a good starting point for promoting interdisciplinary thinking and action in the context of ESD, also regarding task design.

The requirement to design an ESD-MM task offers the PSMTs the opportunity to learn in this sense from a scientifical, didactical and interdisciplinary perspective to test their own contributions to transformative education. In Wiegand and Borromeo Ferri (2023) the structure of the integrative ESD modelling seminar is described. In this article, we emphasize the part in which the PSMTs are asked to develop their own ESD-MM task. Referring to UNESCO (2009) and several authors, e.g., Rieckmann (2012, 2021) or Singer-Brodowski et al. (2025), we educate our PSMTs to become 'change agents', thus sustainability professionals who are specialized in promoting and implementing sustainable practices within various fields and in our case, in mathematics (education).

To create transparency regarding the knowledge that the PSMTs will gain on SDGs and ESD, which cumulatively in the task design process, here are a few key aspects in **Table 1**. This content was continuously applied in the context of MM during the semester. By means of suitable tasks and example topics, students are made aware that understanding sustainable action is a matter of perspective and depends on one's living situation or place of residence. In this way, ESD-MM mathematics trains the ability to adopt different perspectives. The focus of ESD-MM task is on raising awareness of different living situations and interdependencies around the world.

Providing PSMTs with such knowledge seems justified and necessary. When Vásquez et al. (2020), in their qualitative study of ESD-beliefs asked early childhood and primary PSMTs, if they felt well prepared to teach ESD, their results were, that of the 136 respondents, 78% answered "no". Current results of a mixed-methods-intervention study with primary PSMTs attending the ESD-MM seminar presented in this paper show significant improvement in PSMTs' knowledge of theoretical and practical knowledge regarding ESD/SDGs and MM (Bulut & Borromeo Ferri, 2025).

Table 1. ESD/SDG content of the ESD-MM seminar

Central content for SDG and ESD framework for PSMTs in the ESD-MM-seminar
Introduction to sustainable development cooperation: Agenda 21 and the sustainable development model
The 2030 agenda with the global networking of the 17 SDGs and the interdependencies of (world) societies: Development of the network of interrelationships and different perspectives in dealing with global change as well as sustainable development as a mandate for society as a whole using the example of e-mobility as a building block of the German energy transition and its dependence on aluminum mining in Brazil.
Exemplary presentation of modelling tasks on the topic of e-mobility with a subsequent discussion on the sustainability of the e-mobility transition.
Clarification of the educational goal of ESD: • "Gestaltungskompetenz" (design competences) and the triad "recognize, evaluate, act" by de Haan (2002). • ESD 1 (instrumental ESD) as an educational concept that teaches knowledge, values and lifestyles that contribute to transformation. • ESD 2, emancipatory ESD, trains autonomous action for controversial discourse and critical reflection on social models, norms and values. In terms of ESD 2, people should be empowered to think critically about sustainability and learn lifestyles and behaviors for a sustainable future in ESD 1 (Rieckmann, 2021).
Clarification of the design-criteria for the ESD-MM task to be developed. Deepening theoretical knowledge of SDG/ESD and MM.

Table 2. ESD-MM design-criteria (modified version of Wiegand, 2024)

	Criterion	Explanation
1	Dependence of the ESD framework and concept of MM	Both concepts, modelling and ESD, are considered in one task. In particular criteria for modelling problems (Maaß, 2007; UNESCO, 2020; Wiegand & Borromeo Ferri, 2024).
2	MM and interdisciplinarity as a bridge between the learning content of mathematics and sustainable/non-sustainable issues	The task offers the possibility for interdisciplinary thinking and action from various perspectives. MM serves as a bridge between the learning content of mathematics and (non) sustainable development processes (Borromeo Ferri & Mousoulides, 2017; English, 2009).
3	A division of the task into a mathematical-factual and an ethical-moral part	Answering or discussing the sustainability issues of an ESD topic based on mathematical ways of thinking and working requires a corresponding division of the task into two parts. In the first part, the posed problem enables the handling of data and facts so that it leads to a mathematical result. A further question in the second part should be formulated in such a way that the individual is approached from an ethical and moral point of view, in order to formulate its own visions for action. These can in turn be consolidated with the help of mathematics (de Haan, 2002, 2010; UNESCO 2020; United Nations, 2015)
4	"Me" reference within glocal and lobal	Rethinking one's own role in society and, on the other hand, recognizing responsibility for the "one world". In the task the "me" reference becomes visible, either from local to global or global to local issues in the real problem situation (United Nations , 2015).
5	A methodological-didactic concept of social learning	Either formulated in the task itself or in the corresponding lesson plan, social forms of learning as should be promoted in the working phase. Discourse about one's own actions and the actions of others provides an opportunity to engage with different subjective interpretations of the world (de Haan 2010; UNESCO, 2020).

Task Design for Sustainability Issues: Design-Criteria For ESD-MM Tasks

As presented before, the knowledge about SDGs, ESD and modelling are connected and was concretely applied by the PSMTs for the ESD-task design activity.

Appropriate mathematical task formats can be used to impart "knowledge" about the sustainability of actions and about global and local developments. A decisive contribution is the provision of content-orientated and cognitively activating, integrative tasks (Blum, 2015; Watson & Ohtani, 2015). Task development for sustainability issues is currently being discussed in mathematics education. Coles (2025) for example provides a theoretical framework for the socio-ecological task of promoting transformative learning. Although results of studies show, that in primary and secondary mathematics textbooks sustainability themes across mathematics topics became visible (e.g., Kim & Pang, 2022; Vásquez et al., 2021), there is a strong lack of evidence-based tasks to engage learners in critical thinking about social, economic, and ecological issues through mathematics. From the authors perspective, ESD-MM tasks can be one possibility for teaching sustainability issues from a mathematical perspective in school and teacher education. In the following the developed design-criteria from Wiegand (2024) are briefly described (**Table 2**).

First, in the sense of integrating ESD and mathematics, especially MM, it is important that both concepts, modelling and ESD, are considered in one task. Regarding the characterization of ESD (de Haan, 2002; United Nations, 2015), one must consider which SDGs or perspectives of the chosen sustainability topic invite critical reflection. Finally, consideration must be given to whether the well-considered question for example of a PSMT from our study, 'What impact does the use of disposable e-cigarettes have on lithium waste, waste generation and water consumption in Germany?' fulfils the criteria of a modelling problem (Maaß, 2010). Having this in mind, the first criteria is the "dependence of the ESD framework and concept of MM".

MM problems often provoke interdisciplinary ways of solving (Goos et al., 2023). Questions of sustainability issues can be very complex for solving and thus experts from several STEAM-fields are needed (Wiegand & Borromeo Ferri, 2023). The interdisciplinary nature of the ESD-MM task requires mathematical results from the various disciplines to be considered, evaluated and reflected upon. Taking the e-cigarette consumption mentioned as an example, this means, e.g., recognizing and discussing the consequences of lithium mining for the local population, the ecosystems, but also the economic developments and the political handling of consumption from, e.g., a European perspective. Thus, the second criteria is named "MM and interdisciplinarity as a bridge between the learning content of mathematics and sustainable/non-sustainable issues."

Working mathematically makes it possible to obtain results in a wide variety of forms, such as simulations based on numerical methods or big data. The modelling process of the above-mentioned lithium waste question, based on given information/data for the learners, offers and covers several mathematical topics, like percentage and measurement calculation or dealing with data itself. If a learner has modelled this task and gets a result that the lithium deposits in the country of origin (e.g., Chile) require approximately 48,000,000 liters of water per year, then the task is not "solved". We are in the context of ESD and modelling, and this mathematical fact must first be interpreted and validated both mathematically and regarding sustainability issues. Teachers as 'change agents' should challenge these mathematical results in both an ethical and moral way (United Nations, 2015) directed personally to themselves and their learners in school. In this context, it is important to consider the living conditions of people living in Chile and affected by lithium production, as well as to discuss the European attitude towards the consumption of disposable products, as the following question requires: "Based on your modelled results, what could be the reasons why the European Parliament have not yet taken measures to ban these disposable products? Please argue mathematically along with possible data. Would you like to

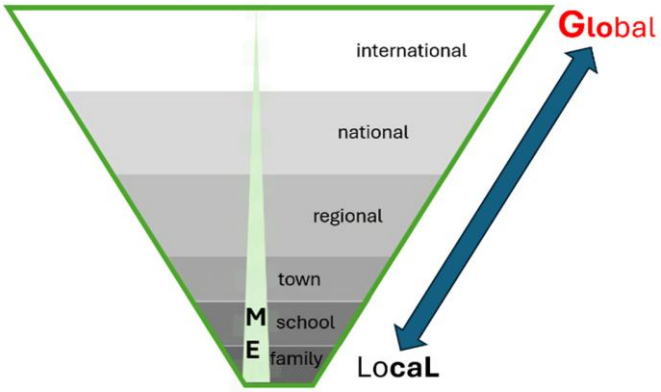

Figure 1. Multi-level model of the "me-perspective"–Multi-perspectivity of ESD (Adapted from United Nations, 2015)

demand a ban?" Thus, the third criteria is to "divide the task into a mathematical/empirical-factual and an ethical-moral part." Teachers and learners are invited to learn to use MM as a basis for argumentation in sustainability discourses, which identifies (decision-making) dilemmas or conflicts (de Haan, 2002, 2010).

The fourth design-criteria can be seen as a specification of the third criteria, but it has a unique position due to its importance. Although the criteria is named "me reference within glocal and lobal" it is about reflecting on one's own perspective and actions, but in the sense of a change of perspective from the local to the global level and vice versa. The following multi-level model in **Figure 1**, adapted from United Nations (2015) by the authors to the ESD-task context, illustrates how this "me-perspective" is anchored in the design-criteria of ESD-MM tasks in a wider context.

With this model, the authors refer to the agreement of the 2030 agenda on development for society as a whole: "Governments as well as parliaments, the United Nations system and other international institutions, local authorities, indigenous peoples, civil society, business and the private sector, the scientific and academic community–and all people" (United Nations, 2015). The participation in the 'transformation of the world' can be initiated by individuals (micro level) and lifestyle groups (meso level), as well as by international operating systems (macro level). Both initiatives,

(1) the lobal–starting from the self and impacting on social change–or

(2) the glocal–starting from society as a whole and targeting the individual–require knowledge of the SDGs and their interconnectedness (United Nations, 2015).

The PSMTs as the task creators can ask themselves the following question: "Does my ESD-MM task address a sustainable issue, e.g., the working and living conditions of people in other cultures, thus highlighting a comparison with one's own life situation?"

Through the fifth criteria "a methodological-didactic concept of social learning" the ESD-MM task should be embedded regarding lesson planning. Either formulated in the task itself or in the corresponding lesson plan, social forms of learning should be promoted in the working phase.

Finally, an overview is given in **Table 2** regarding the design-criteria and their explanations in a modified version of Wiegand (2024).

METHODOLOGY

Analyzing how the task development was approached and how the tasks differ called for a qualitative study design (Creswell, 2008) and specifically the method of type-forming qualitative content analysis, according to Kuckartz (2016) and its steps (**Table 3**) was used.

Sample and Data Collection

The sample comprised 35 PSMTs for secondary school with an age between 19-21 years for the **RQ1**. The PSMTs were between the 1st and 2nd year of study and attended the ESD-MM seminar led by both authors. This seminar took place parallel to the compulsory practical semester, while PSMTs teach in the school for six months.

The sample consists of four cohorts of PSMTs from the years 2022-2024. 10 PSMTs from the fourth cohort were trained explicitly in using and applying the design-criteria and thus the **RQ2** is focused on this group. All PSMTs agreed to take voluntarily in this study and that their ESD-MM tasks and the associated written explanations can be used for this study anonymously. Participants could withdraw from the study at any time.

The ESD Modelling Seminar–Basis For the Task Development and Study Design

The ESD-MM seminar is described in detail in Wiegand and Borromeo Ferri (2023). This seminar is the basis of the task development process and builds the intervention part of the study design with several cohorts followed by the submission of the PSMTs individual final ESD-MM tasks eight weeks after the end of the seminar. Briefly summarized, the seminar takes place over a whole semester of five month. It is divided into 4 units for 4 hours each, which means one unit is 240 minutes.

Table 3. Steps of the qualitative type-forming content analysis (Kuckartz, 2016)

	Step
1	Determining the meaning, purpose and focus of typing.
2	Selection of the relevant dimensions of type formation and determination of the feature space.
3	Coding or recoding of the selected material.
4	Determination of the method of type formation and construction of the typology.
5	Allocation of all cases of the study to the types formed.
6	Description of the typology, the individual types and in-depth individual case interpretation.
7	Analysis of the relationships between types and secondary information.
8	Complex relationships between types and other categories.

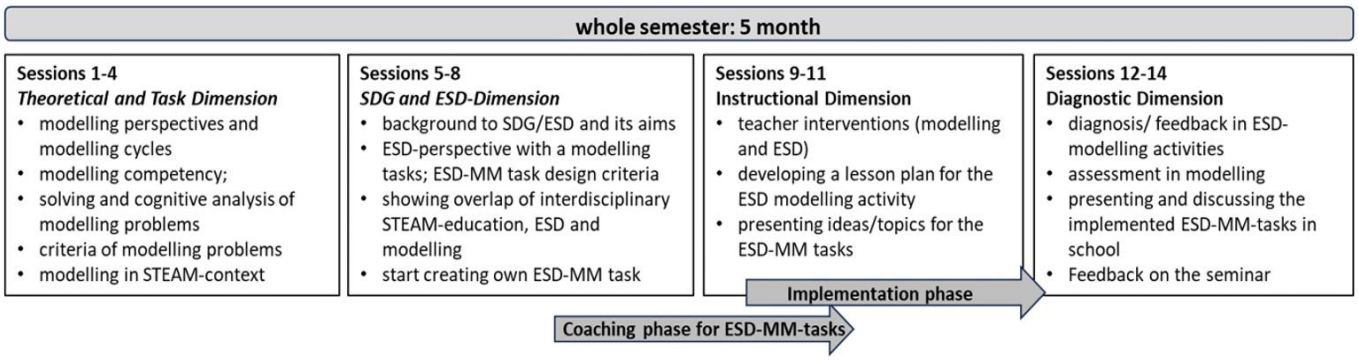

Figure 2. ESD modelling seminar (see also Borromeo Ferri & Blum, 2010; Wiegand & Borromeo Ferri, 2023)

The seminar, shown in its structure in **Figure 2**, is based on the dimensions of teacher competencies for modelling according to Borromeo Ferri and Blum (2010) and Borromeo Ferri (2018) as well as the new integrated SDG/ESD-dimension (unit 2). The units on MM are carried out by the second author. The ESD units were designed and implemented by the first author.

The selected content activities show the implementation of the integrated approach of ESD and modelling in the seminar: Prepared from unit 1, the introduction to the design-criteria of the ESD-MM tasks and the task development based on this begins in unit 2. First, the PSMTs develop initial task ideas which they present in the seminar and receive feedback from the participants and the authors. It is important to emphasize that the PSMTs were free to choose the topic, the link to the SDGs, the mathematical content and the grade level according to their interests. The development process is accompanied by coaching from the authors and continues for 3 weeks until the final version of the task is completed. The aim was to ensure the quality of the developed tasks, also regarding the use criteria. In unit 4, all PSMTs present their experiences with their task in school and reflect on them.

Data Collection

The PSMTs had to submit an individual written reflection based on guidelines eight weeks after the seminar. This had to include, among other things, the final ESD-MM task with accompanying commentary and associated materials (texts, videos, task sheets, etc.) and a "sample solution", insofar as this is possible. The lesson plan and task material together with the ESD-MM tasks is named as "a set" and has an average length of 2 pages each. This results in a total of 140 pages of text material for the 35 ESD-MM tasks. Referring to a similar procedure of data collection by Wiegand and Borromeo Ferri (2023) and according to Heinze (2016), these text materials are '[…] scientific descriptions of a social object. They are neither "pre-knowledgeable", like everyday descriptions, nor scientific-quantitative, i.e., not in the form of frequencies, quantities, series of numbers or indices that have to be explained or interpreted by the researcher. Rather, they are meaningful in themselves' (Heinze, 2016, p. 13). Thus, for the analyses, not only should the pure formulation of the task be used, but the selection of additional materials. In doing so, we are following Watson and Ohtani (2015), who emphasized that a learning task is always integrated into a comprehensive teaching-learning process.

Data Analysis

The type-forming qualitative content analysis according to Kuckartz (2016) has the goal of methodological control and transparency and forms the basis for the data analysis. The type-forming process has 8 steps, see **Table 3**.

In the following these steps are introduced by connecting them to our study: The focus on type-forming, step 1, results from the RQs of the study, because the aim is to contrast the ESD-MM tasks to identify differences and similarities. The relevant dimensions, see step 2, were developed in a theory-based manner, already described before:

(1) the MM multi-perspectivity and

(2) the ESD multi-perspectivity.

These dimensions open the feature space. According to Mayring (2022), the necessary describing characteristics of each dimension arise, as also in our case, on the one hand from the successive data analysis, and, on the other hand, they are compared with the theory again, see **Table 4** and following explanations.

Dimension of the MM multi-perspectivity

Characteristic I: Within the framework of a less complex task format, learners are encouraged to discuss ESD issues using researched or given data and facts in their modelling process. The aim of the standard task type is to understand MM and its parameters and to use them to design and solve (individual) sustainability discourses up to a global level. The task fulfils requirement level I with the simple application of mathematical methods.

Characteristic II: Topics of sustainable and non-sustainable development offer ideal starting points,

Table 4. Dimensions and characteristics for the type-forming procedure

Dimensions	Characteristics
MM multi-perspectivity	1. Standard task for the use of ESD-related data materials 2. Meaningful application of mathematical methods in the ESD context 3. Context-emphasised mathematical methods as a tool for achieving ESD goals
ESD multi-perspectivity	Glocal approach: Focus on the global development of a decision dilemma sufficient to the local situation (glocal) and on the formation of an individual decision dilemma sufficient to the global situation (lobal)

Table 5. Part of the coding scheme for case locating (example of characteristic III with [I] in the lithium waste/vape task)

	Coded parts of the lithium waste/vape task (table)	Description/coding rule for characteristics
Characteristic III: Context-emphasized mathematical methods as a tool for achieving ESD goals	What impact does the use of disposable e-cigarettes have on lithium waste, waste generation and water consumption in Germany? • Documentary short clip about lithium mining • Information sheet on lithium wastage in disposable e-cigarettes	In connection with the materials and information provided, the task is requirement level III. The modelling process requires a complex connection of interdisciplinary information provided or researched by the learners.
Characteristic I: Glocal approach	Based on your modelled results, what could be the reasons why the European Parliament has not yet taken measures to ban these disposable products? Please argue mathematically along with possible data. Would you like to request a ban?	The task formulation offers explicitly to address the specific life situation of the learners and broadens the view to the glocal level based on modelling. Task formulation requests a strong discourse that relate to one or more of the three dimensions of sustainability (social, ecology, economics) as well as inter- and intragenerational justice

Table 6. Part of the coding scheme (example of approach/idea for the lithium waste/vape task)

	Coded part for the approach/idea for task development (here the case of ACD)	Description/coding rule for the approach/idea for task development for ACD and MCD
Approach/idea for the task development process	"At the school where I teach, there is a significant problem with disposable cigarettes/vapes. The idea for this task arose to draw attention to this problem and to raise awareness […]. The task does not allow for any direct solutions, which ensures the openness of the task. This adds a complex dimension to the task, as several assumptions must be made and researched, which means that the modelling cycle must be run through several times […]." (PSMT_32)	If the idea for the ESD-MM task is formulated in such a way that it comes from the direct everyday life of the PSMTS or from teaching at school, it is coded as ACD. If the idea for the ESD-MM task is formulated in such a way that it relates to the mathematical topic previously or currently being taught in class, it is coded as MCD. (The left was coded as ACD)

motivation and illustration of mathematical thinking and working methods. In this type of task, the focus is on the mathematical knowledge acquired, which is used to solve an appropriately designed ESD-MM task. The aim of the meaningful task is to practice and deepen mathematical procedures in application. At the same time, the meaningfulness of mathematics should be conveyed to the learner and mathematics should be made more attractive, motivating and accessible. The meaningful task fulfils requirement level II with the more complex steps of modelling.

Characteristic III: The non-mathematical question within the sustainability context is seen as a work assignment and starting point and for modelling activities. MM, together with mathematical methods, contributes to the clarification of ESD issues in an interdisciplinary manner. The aim of the context-related task is to perceive, understand and analyze the problem in its real environment by creatively using mathematics as a tool. This task fulfils the highest requirement level III with the required networked thinking and high complexity.

Dimension of the multi-perspective of education for sustainable development

The cognitive paths through the social levels of action provide a characteristic of an ESD learning task: the glocal approach (see also **Figure 1**): Focus on the global development of a decision dilemma sufficient to the local situation (glocal) and on the formation of an individual decision dilemma sufficient to the global situation (lobal).

The need for action in society in terms of the SDGs is focused on individual action. Following on from this, learners' opportunities to exert influence are developed and their shared responsibility for issues affecting society as a whole is discussed. The focus is on the ability to reflect on and evaluate individual life decisions and their impact on the global situation. Learners are encouraged to project their individual actions onto the level of society.

Following step 3 according to Kuckartz (2016), the ESD tasks and the written material were analyzed and coded discursively by the two authors along the dimensions and characteristics, see **Table 4**. Further coding was done regarding the approach of the tasks along the codes. The validation of the codes was done by using the method of "member checking" (Flick, 2018). Though the discursively coding of the data it was possible to identify patterns and to start with the type-forming (step 4) and then allocating all cases (Step 5). For example, the ESD-task about the lithium waste/vapes shown before should be solved by learners of grade 10. As "the set" was analyzed, the goal of this task was to address the specific life situation of the learners and broadens the view to the glocal level through modelling activities. This combines the characteristic I, as this is always included, but then characteristics III and type-formed this task within the named type as "mathematical discourse tasks for ESD reflection". **Table 5** provides a part of the coding scheme for the lithium waste/vape task.

For the analysis, where the idea for the topic approached, this was coded in the written reflection during the analysis process between step 5 and step 6. In **Table 6** a part of the coding scheme becomes visible.

The description of each type (step 6) with a prototype is presented before and due to this also the relationships (step 7) between types and other categories (step 8) become visible.

RESULTS

The presentation of the results follows along with the posed RQs.

Table 7. Approaches to the task development process

MCD	ACD
"Since the class was currently learning about geometric solids and had covered cylinders in the previous lesson, I decided to focus on the consumption of cans. So, I thought about how I could link geometric solids, in this case cylinders, to a modelling task and to sustainable development. I decided to look at the consumption of aluminum cans and the bauxite needed to make them."	"I chose the topic of CO_2 because I believe that everyone has heard of it and will be affected by it sooner or later. That's why it was important to me to develop a modelling task in this context. My goal was to develop a modelling task that not only offers mathematical concepts and skills but also establishes a direct link to the real world, thereby raising awareness of environmental issues and sustainability."
"My sixth-grade ESD-MM task is about water consumption in everyday life and is linked to the issue of water scarcity in the world. To solve this task, a lot of multiplication must be done mathematically. The current topic in class is the greatest common divisor, least common denominator and fractions."	"At the school where I teach, there is a significant problem with disposable cigarettes/vapes. The idea for this task arose to draw attention to this problem and to raise awareness [...]. In addition to modelling the amount of lithium waste generated by disposable cigarettes and vapes, the task also poses further questions. These include the water consumption involved in lithium mining in the country of origin and the contribution of young people to the generation of waste. This gives the task a complex dimension."

RQ1

Based on the analysis of 35 PSMTs written reflections, two approaches for the task development process could be found:

(1) mathematical content driven (MCD), and

(2) actual context driven (ACD).

The MCD approach means that the idea for the ESD-MM task was derived from previous or current mathematical content taught at school. 10 of the PSMTs, who preferred this approach, considered which sustainability issues could be addressed in a way that would foster specific mathematical content through modelling to highlight the importance of mathematics in the context of sustainability for the learners in school.

The main part of the PSMTs, 25, preferred the ACD approach, that means thinking about a (current) sustainability issue in general or specific events in their schools. These thoughts were then linked to the development of an ESD-MM task and adapted to the class being taught.

The result that students particularly preferred the ACD approach highlighted the importance of interdisciplinary connections. This demonstrates the cross-curricular and everyday-oriented thinking of PSMTs and reinforces the approach of integrating mathematics into teacher training for other subjects such as politics, history, and ethics. Due to the fact, that German teachers have to study two subjects to teach at school, the ACD method holds particular relevance in teacher education.

The following quotations in **Table 7** illustrate the two different approaches.

Designing the ESD-MM tasks was equally a learning process and a challenge for the PSMTs as formulated here:

> Developing an ESD-MM task was a challenge for me. In the seminar and in view of my work with modelling tasks, I realized their importance and effectiveness in promoting numerous skills in learners. Furthermore, the compatibility of modelling tasks and ESD became apparent to me, as did the opportunity to educate learners in a sustainable and environmentally conscious way at an early stage.

The PSMTs were able to reflect themselves also based on the feedback of the authors as the teacher educators. The following quote illustrates a motivated PSMT, because of the challenging task design with the criteria paid off:

> Overall, developing and implementing my ESD-MM task taught me the importance of linking mathematical concepts and skills to real-world applications while promoting awareness of sustainability issues [...]. I was able to spark the learners' interest and motivate them to engage with sustainable consumption.

RQ2

ESD-MM task development played a significant role in the professionalization of PSMTs.

The ten tasks demonstrated that the design-criteria for developing an ESD-MM task are helpful. The research findings show that the design-criteria can be applied at different ability levels for teachers and learners. Following the criteria, ESD-MM tasks could be developed and implemented with varying degrees of difficulty, time frame, and focus on specific competencies. The design-criteria and the approach described can serve as a guide for teachers in other cultural and educational systems/curricula as well. As the criteria are formulated in a very open way, they can be transferred to other teaching content. The demands of the United Nations to integrate ESD into mathematics teaching are thereby substantiated.

The analysis of the ten ESD-MM tasks developed by the PSMTs, who were explicitly trained in the design-criteria for task development, show that all tasks have a real-life connection to sustainable and non-sustainable development processes. The generational justice discourse and at least one of the dimensions of sustainability (ecology, economic, social) is focused on and discussed in relation to the other dimensions. The following quote from a PSMT illustrates the aspiration of his task by reflecting the implementation:

> The learners discovered how various factors, such as the amount of chicken consumed and the emission values, can influence the carbon footprint. This helps them to understand environmental problems not in isolation, but as part of a larger system of ecological, economic and social factors. [...] Especially SDGs 12 and 13 are in the focus.

Table 8. ESD-modelling task types

Dimensions	ESD multi-perspectivity	
	Characteristics	**Glocal approach**
MM-multi-perspectivity	Standard task for the use of ESD-related data materials	*Type 1. Descriptive ESD MM tasks*
	Meaningful application of mathematical methods in the ESD context	*Type 2. Procedural ESD MM tasks*
	Context-emphasized mathematical methods as a tool for achieving ESD goals	*Type 3. Mathematical discourse tasks for ESD reflection*

Table 9. Prototype for descriptive ESD MM task

	CO_2 pollution of meat consumption and meat alternatives (great 8)
Empirical-factual questions	Approximately how much meat is eaten per year in Germany? Calculate how much CO_2 is emitted by meat consumption in Germany per year.
Working materials	• Information material on the meat consumption of the average German per year for pork, beef and chicken • Information materials on CO_2 emissions per kg of meat
Modelling	Creating models of CO_2 emissions from the consumption of meat and meat alternatives
Recognizing, evaluating, acting	• Contribution of German meat consumption to CO_2 emissions • Recognizing that the types of meat emit different amounts of CO_2 • Discourse on alternatives to meat consumption • Assessment of the impact of meat consumption and the alternatives on the Umwelt.
Interdisciplinary knowledge, etc.	• Effects of meat consumption and meat alternatives on the environment, • resource consumption, resource availability • Ethical issues or social aspects: Factory farming and animal suffering • agricultural production of tofu or plant-based meat substitutes • Supply chains
Me-reference	Examination of personal nutrition with a view to environmental protection, animal welfare aspects, among other things
Ethical and moral issues	*Now think about how you could reduce meat consumption. Are there alternatives? But are they more environmentally friendly? And if so, how much CO_2 could be saved by the alternatives?*

Suitable media (e.g., short film clips from documentaries) and working materials (e.g., information texts) are used to build bridges between the levels of action. This methodological-didactical framing of the ESD-MM tasks is used either at the beginning of the lesson to present the global situation or towards the end to place the individual levels of action in the global context, as reflected as follows:

> The fact that valuable resources are being carelessly squandered highlights the unsustainable consumption of products (SDG 12) and inequality (SDG 10) in countries that cannot afford such waste. The short film deals with the initial effects of lithium mining [...].

Regardless of the ways in which knowledge is acquired, the ESD discourses are based on mathematical principles. The tasks address the connection between an individual and events in other parts of the world. They visualize the weighting of the sustainability dimensions in the different living communities.

The PSMTs have planned group work and discussion rounds with a focus on social learning. The aim of all tasks is to recognize sustainable and non-sustainable development processes, evoking decision-making dilemmas and a vision-oriented discussion for an individual contribution to transformation. A PSMT reflected:

> The emphasis on alternatives to meat consumption and modelling the potential CO_2 savings through the use of plant-based meat substitutes made the concrete effects of their actions clear to the learners and encouraged them to make sustainable decisions.

The commonalties in the tasks also show the connections between the "sets" in the sense of the evaluation step 7 and step 8 according to Kuckartz (2016). Nevertheless, differences in the tasks became apparent. In terms of the **RQ2** (a, and b) it will now be shown how these differences look like and how they can be described.

The tasks are categorized as standard, meaningful, or contextual. The combination of the described dimensions MM-multi-perspectivity with ESD-multi-perspectivity as well as the characteristics shown in **Table 8** provides a total of three task types, based on the qualitative analysis. The task types are described using prototypical tasks.

Type 1. Descriptive ESD MM Task

These type 1 tasks use simple models to describe sustainable and non-sustainable situations or developments (see **Table 9**). This type of task meets requirement level I with simple modelling processes. MM structures (non-)sustainable development processes and situations and furthermore forms an easy-to-understand and solid mathematical basis for the discourse on the actual and target states of sustainable action. To solve this task, simple mathematical representations (e.g., tables, diagrams, and statistics) are required, which are interpreted in the context of ESD. The focus is on recognizing, describing and interpreting given information.

The task in **Table 9** on the topic of meat consumption versus meat alternatives is a prototype for this type of task. Based on the modelling of meat consumption in Germany, the associated amount of CO_2 emissions is calculated. The search at possible meat alternatives in the second part of the task invites discussion with the question "Are there alternatives?", beyond the (supposed) ecological advantages of meat alternatives. Simple tables are used to describe individual (non) sustainable life decisions or lifestyles based on facts.

Table 10. Prototype for procedural ESD-MM task

	Resource consumption for lithium extraction in the Atacama Desert (Chile) (grade 9)
Empirical-factual questions	*Determine the area and volume of the lithium basin. Indicates the result in liters (Note. Use both illustrations or Google Maps "Atacama Desert" for the calculation).*
Working materials	1. Information text on lithium mining and its consequences 2. Illustrations of a lithium basin: Trucks standing at a basin and Google Maps recording with "measurement" function
Modelling	Modelling of the basin size and estimating the amount of drinking water used
Recognizing, evaluating, acting	• Huge amount of water are used to extract lithium in one of the driest areas in the world • People in the surrounding areas who are deprived of their livelihood (water) • Discourse on ESD action through the prolonged use or recycling of used electronic devices
Interdisciplinary knowledge, etc.	• Extracting lithium salts from the rock of the desert, the evaporation process of the water environmentally damage the use of water reserves in the desert • Effects on nature and the people living there
Me-reference	Personal reduction of lithium consumption by restricting the consumption of smartphones, laptops and electric cars
Ethical and moral issues	*What are the consequences of the use of drinking water on the people in the surrounding areas? What are the consequences for the environment when drinking water is used? Take the information from the text (Note. Put yourself in the shoes of the people as they lived before and after mining and how it has changed). What alternatives could people use if no raw materials are to be mined for a new battery?*

Table 11. Prototype for mathematical discourse task for ESD reflection

	Consumption of disposable products using the example of disposable cigarettes/vapes (grade 10)
Empirical-factual questions	*What impact does the use of disposable e-cigarettes have on lithium waste, waste generation and water consumption in Germany?*
Working materials	• Short film about lithium mining • Information sheet on lithium wastage in disposable e-cigarettes
Modelling	Modelling of lithium wastage due to the consumption of e-cigarettes per year
Recognizing, evaluating, acting	• Waste of resources disposable cigarettes/vapes • Negative health effects due to the ingredients of e-cigarettes • Environmental impact of production, use and waste disposal
Interdisciplinary knowledge, etc.	• Lithium mining • Health effects of the consumption of e-cigarettes • Recycling options
Me-reference	Recognizing the effects of e-cigarettes and reflecting on your own consumption
Ethical and moral issues	*Based on your modelled results, what could be the reasons why the European Parliament has not yet taken measures to ban these disposable products? Please argue mathematically along with possible data. Would you like to request a ban?*

The task can be transferred to the general discourse of different dietary styles. For example, the consumption of virtual water by different foods should also be mentioned here. In both cases, the bridge to the import of food from other countries and the associated effects can be addressed.

Type 2. Procedural ESD MM Task

This type 2 of task fulfils requirement level II of modelling, is mathematically motivated internally and uses the contents of an ESD topic to give mathematics a reference to reality, a reason, a motivation or "meaningfulness". The application serves to practice and deepen mathematical methods and procedures.

The prototype in **Table 10** on the topic of lithium mining in the Atacama Desert in Chile ties in with the unit areas and volumes that the middle school students had dealt with in class shortly before–the formulas are applied to a real situation. In the second part of the task, lithium mining and its consequences are discussed independently of facts, i.e., the calculated data have no concrete influence on the course of the ESD discourse. This distinguishes this type of task from the first and third types of tasks, in which the facts are the basis of a sustainability discourse and action.

The topic of lithium mining in the desert in Chile is prototypical for the controversy of protecting the habitat and the intensification of resource consumption.

Type 3. Mathematical Discourse Task for ESD Reflection

In a contextual ESD-MM task, the ESD content is the focus of the mathematics lesson, and the mathematics is used to describe complex or unknown ESD issues. This type of task fulfils requirement level III of modelling. This type of task strongly focuses on discourses that relate to one or more of the three dimensions of sustainability (social, ecology, economics) as well as the generational justice concept.

ESD-MM task type 3 in **Table 11** addresses explicitly the specific life situation of the learners and broadens the view to the glocal level based on modelling. The prototype presented here refers to the resource management of lithium in one's own country or other countries as an example and can focus on plenty of SDGs, e.g., SDG 8, decent work and economic growth or SDG 6, availability of clean water. Finally, the question of a ban on disposable e-cigarettes is also raised, thereby focusing on political options for action. These three exemplarily ESD-MM tasks out of 10 PSMT show that the application of using and fulfilling the same design-criteria by the PSMTs has led to activities that can combine mathematics and sustainability issues in a broad variation and with different foci. **RQ2** can therefore also be answered. Based on the empirical findings, task types have been identified that offer orientation regarding different goals in the field of tension between ESD and mathematics education.

Table 12. Further exemplary task

	The conservationists' fight against a wind farm in Reinhardswald in Hesse (Grade 5)
Empirical-factual question	1) How large must the deforested area be to erect the 18 wind turbines? 2) How many trees could have stood on this area? 3) How many tonnes of CO_2 could the trees on the deforested area have stored in one year? 4) How many tonnes of CO_2 can wind turbines save compared to gas-fired power plants?
Working materials	• Video: Documentation on the planned wind farm • Information on the tree/forest: diameter of a tree, CO_2 uptake per hectare of forest area in tonnes • True-to-scale illustration of a wind turbine next to a tree • Documentation on Reinhardswald: The conservationists' battle
Modelling	Modelling of the areas required for 18 wind turbines; Modelling the number of trees on this area.
Recognising, evaluating, acting	• the loss of trees as CO_2 reservoirs, • Saving CO_2 through the use of wind turbines to generate energy • Over a longer period of time, wind turbines contribute to reducing CO_2 in the atmosphere to a greater extent than the storage capacities of trees on the same area can achieve. • Renewable energy is not sustainable per se.
Interdisciplinary knowledge, etc.	• Importance of the forest for CO_2 storage • Importance of mathematical relationships as a basis for political decisions • CO_2 neutrality of wind power plants compared to gas-fired power plants, among others
Me-reference	Recognising that learners need to be mindful of their electricity consumption. Importance of the forest beyond the tree as a CO_2 store.
Ethical and moral issues	Can deforestation in the Reinhardswald be justified based on the modelled positive CO_2 balance of wind turbines?

To illustrate the diversity of PSMTs topics, a different task is presented in **Table 12**. The task "conservationists' fight against a wind farm in Reinhardswald in Hesse" is a prototype for the targeted use of mathematical content to illustrate and resolve controversies such as the destruction of CO_2 storing forest areas for the construction of (CO_2 neutral!?) wind turbines. (Note: Reinhardswald is a name of a forest close to the city of Kassel).

In the following, we discuss the significance of the ESD-MM task design for teacher professionalization. Furthermore, we would like to highlight the potential of the ESD-MM tasks based on the design-criteria for the teaching and learning sustainability issues in schools.

DISCUSSION

In this study four cohorts of PSMTs were investigated in an ESD-modelling seminar in two years, of whom one cohort received explicit training on the ESD-MM criteria. Both with and without this training, a central goal was that PSMT learn how mathematics can be the basis to describe, solve and understand sustainability issues for themselves and in their future work in the classroom by specifically developing and implementing ESD-MM tasks. The integrated approach of ESD and modelling realized in the seminar professionalized the PSMTs to such an extent that they were able to successfully develop. The results show, that PSMTs approached in a mathematical and content driven way for developing task for the learners and that finally -regarding the subgroup that was explicitly given design-criteria as a guideline–three ESD-MM task types could be found empirically.

The research results provide a framework for the design of ESD-MM tasks, demonstrating that the design-criteria can be applied at different levels of ability for both teachers and learners. Following these guidelines, ESD-MM tasks of varying levels of difficulty, time requirements and different focused competencies can be developed and implemented. The demands of the United Nations (2015) to integrate ESD into mathematics lessons are thus concretized for various educational sectors of secondary and higher education.

All PSMTs have become aware of the importance of mathematics for promoting sustainability issues. These results consist with research emphasizing the transformative potential of targeted interventions in sustainability education (Alsina & Vásquez, 2024; Nguyen et al., 2020). Teachers are the key multipliers in giving young people the opportunity to reflect on sustainability issues and to use mathematics as an important and necessary discipline for finding solutions. In their current study, Guerrero-Ortiz et al. (2026) emphasize the importance of teacher training courses in order to identify the sustainability competencies that emerge in teachers' relationship with mathematical abilities. The researchers' findings highlight the importance of generating more activities that reinforce the anticipatory competency in pre-service teachers for future sustainability scenarios. In view of these findings it supports the fact that our ESD-MM seminar with the integrated approach was conducted and evaluated over a longer period. This makes it more valid for using its structure and contents in the mathematics education community for further research in teacher training.

In further studies, it would be interesting to investigate the development process and finally the quality level of ESD-MM tasks by comparing two groups of PSMTs using and not using the design-criteria in a controlled design, which was not the intention in the study presented here. However, more questions arise: Would PSMT have major problems in bringing together ESD and modelling independently? Would the tasks be suitable for learners to recognize the importance of mathematics in the context of sustainability and will the tasks promote decision making dilemmas and critical thinking? The effect of this kind of interventions on the learning process could be examined in more detail in the future. Although the PSMTs used the same criteria, three different types of ESD-MM tasks were identified. On the one hand, this is a research result, which shows interestingly the nuancing of the underlying

design-criteria in their applications. On the other hand, these types and the concrete representation here offer a direct possibility for practical implementation. Thus, these ESD-MM tasks can be used for investigating, for example, how learners' awareness of sustainability issues by using mathematics is increasing when working on several tasks.

MM was chosen as the practice in this study and explicitly combined with ESD in teacher training. The interdisciplinary nature of modelling encouraged the PSMTs themselves to think critically and they were also able to encourage their learners. This aligns with finding of Alsina and Vásquez (2024) who emphasized the importance of modelling in promoting systemic thinking among students, to fulfil the SDGs.

In our study the focus was on ESD-MM task, thus promoting MM as a practice. This was realized and reflected by the PSMTs:

> Due to the many links, sustainable development can be implemented very well with the help of MM and included in mathematics lessons. By combining both concepts, values, skills and competencies such as sustainable thinking, responsible action and participation in social decision-making processes can be taught.

We also believe that concrete examples of tasks are necessary to (further) strengthen the subject of mathematics in the context of ESD. Only when teaching materials are available, they can be used by teachers in practice. Borromeo Ferri (2025) has already shown in a study involving 105 secondary school students that ESD-MM tasks significantly improve young people's environmental awareness after just one task. In addition to the findings, there are limitations of the study. The sample of 35 PSMTs seem less, but several seminars had to be conducted to get finally the participants in order to have an adequate data basis for our qualitative study. For RQ 2 only ten PSMTs were trained and we get ten ESD-MM task, we analyzed accordingly in detail. Our results give us helpful insights into further studies with a larger sample. Furthermore, due to the focus on pre-service secondary mathematics teachers. It is therefore not possible to make any statements about differences in the ESD-MM tasks developed by high school or primary teachers. However, this offers a further field of research in ESD-MM task development, as the target groups for the tasks may then differ.

CONCLUSION

With the ESM modelling seminars for PSMTs, which have already been held several times and are currently ongoing, we are promoting to train them to SDG 'change agents'. As the teacher matters most (Hattie, 2008) in order that ESD and mathematics are taught integratively in school, the education of the PSMTs is important. To highlight how the PSMTs were sensitized by giving them a foundation for further implementing sustainability issues integrating mathematics has been formulated by one of them:

> I was able to take a lot away from the seminar. Before, I hadn't realized how well ESD fits into maths lessons […] This has heightened my awareness of the complexity of global challenges and shown me how important it is to prepare young people to make responsible and sustainable decisions.

This is a promising example, that further research in the direction of ESD in mathematics education must proceed to offer, beside evidence-based results, also proven best-practice examples for school implementation.

Author contributions: SW & RBF: data analysis; **SW:** ESD study conception: **RBF:** Mathematical modelling course conception; **SW & RBF:** design, literature search, material preparation, writing – original draft, writing – review & editing. Both authors agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that no ethical approval through a committee was not necessary. The pre-service teachers participated in this study voluntarily and released their data for research purposes in anonymised form. The participants from each of the four cohorts gave their verbal and written consent for the study. The participants were able to withdraw their participation at any time.

AI statement: The authors stated that DeepL were used to improve the clarity of the English language in the article. No content was generated by artificial intelligence. All content was reviewed and verified by the authors.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from corresponding author.

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